Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 3, 1998

MAGNETOSONIC ANALYSIS AND THE METHOD FOR DIAGNOSTICS OF EXPANSION OF A PLASMA CLOUD IN A MAGNETIZED BACKGROUND

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UDC 533.95

The problem of sub-Alfvén expansion of a superconducting plasma sphere in a homogeneous magnetized background is considered. The specifics of a self-consistent model of a low-frequency linear MHD *approximation that we used in the present paper is the simultaneous allowance for the energy necessary for maintaining the field and plasma equilibrium at a moving boundary and the additional perturbation of a decelerating field generated by the currents induced in a background plasma. This has allowed us to clarify significantly the dependence of the radiated magnetohydrodynamic energy on the Mach–Alfvén number. We found and calibrated universal dynamic characteristics on the basis of which we developed new techniques for determining the initial energy and the velocities of expansion of an explosive plasma cloud with the use of the peak values of magnetic signals in the near (quasistatic), transient, and wave zones. The possibility of effective application of these techniques in experiments on laser-plasma cloud generation in a vacuum homogeneous magnetic field is shown.*

Introduction. The conditions under which the expansion of a plasma cloud produced by a space explosion occurs [1] and also the conditions under which laboratory experiments with a laser plasma in magnetic fields [2] occur, can assume the presence of a magnetized background, which should lead to marked variations in the shape and quantitative characteristics of magnetic signals recorded far from the cloud in comparison with the vacuum case [3]. For analysis of the influence of a background plasma on the expansion dynamics, it is convenient to use a low-frequency linear MHD approximation [4], in which the magnetosonic mode of perturbations generated in the medium is fundamental.

With a view for developing the magnetoprobing method of determining the explosion parameters, we consider, within the framework of the above-indicated approximation, the known problem of expansion of a superconducting plasma sphere in a homogeneous medium with a magnetic field B_0 for small Mach–Alfvén numbers $\beta = V_0/V_A < 1$ (V_0 is the initial velocity of the sphere and V_A is the Alfvén velocity in the background). At present, the interest in this problem, which was posed as long ago as during the first high-altitude explosions, is given a new impulse in connection with the formation of a new direction of research concerning the possibility of application of explosive methods to protect the Earth against asteroids [5, 6].

Considering MHD-wave generation by an ideally conducting sphere which first uniformly expands in the background with velocity V_0 and then instantly stops, Lutomirsky employed a low-frequency approximation [4].

Belov et al. [7] found MHD perturbations in a homogeneous background by a special law of the rate of variation in the sphere radius in the form of a quadratically decreasing function of time. In [8], Belov et al. calculated the approximate shape of the deceleration boundaries of a plasma in a high-altitude explosion in the self-consistent formulation of the problem with allowance for scraping of the background particles. However, in our opinion, the energy required to maintain the dynamic plasma and field equilibrium was not included in the energy balance in [8].

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Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 3, pp. 3–13, May–June, 1998. Original article submitted August 19, 1996.

0021-8944/98/3903-0325 \$20.00

Gorbachev [9] obtained analytical solutions for magnetosonic perturbations and Alfvén waves generated by them in the approximation of small values of the magnetic Reynolds number, i.e., for a cloud with a magnetic field completely penetrated into it and finite conductivity. Such an approach can be justified at the late stage of expansion of an explosive plasma cloud rather than at the initial stage, which has a strictly expressed diamagnetic character and to which a "superconducting" model corresponds to a greater extent.

In describing the cloud dynamics at the stage of its expansion, where the cloud still has a spherical shape, the approach that we consider is close to that used in [10]. The basic difference is that, in [10], the decelerating fields at the plasma boundary were assumed to be such as that in vacuum, although the problem of the determination of perturbations under background conditions was solved.

Thus, despite the fact that the problem in the simple formulation indicated above has already been investigated by many authors, it is not yet solved completely, because all necessary conditions for the field and the moving plasma have not been taken into account simultaneously. No exhaustive recommendations for practical use of the results of theoretical and numerical analyses have been given.

The aim of the present work is a step-by-step account of the self-consistent character of motion in calculating the shapes of magnetic signals under the conditions of low-frequency perturbations of a homogeneous background medium at the diamagnetic stage of plasma expansion in order to obtain new methods of recovering, on the basis of these signals, the initial data on the energy $\mathcal{E}_0$ and velocity V_0 of the plasma products of an explosion (their mass M_0 is estimated on the basis of the found $\mathcal{E}_0$ and V_0: $M_0 \sim \mathcal{E}_0/V_0^2$). Since $\beta < 1$, we ignore the effect of background scraping (on the characteristic scale of deceleration, the scraped mass is small compared with the initial mass of the cloud M_0).

Magnetosonic Model of Deceleration of a Plasma Sphere. In the approximation considered, the basic form of MHD perturbations are magnetosonic waves, isotropically diverging from a sphere with velocity V_A. In a spherical coordinate system ρ, θ, φ with the origin in the center of the sphere and the polar axis along the field B_0, the solution of a wave equation, which corresponds to a "magnetic sound," yields the known expression for the vectorial magnetic potential [4]:

$$\mathbf{A} = A_\varphi \mathbf{e}_\varphi = \frac{B_0 R_b}{2}\left[r - \frac{f}{r^2} - \beta\frac{f'}{r}\right]\sin\theta \mathbf{e}_\varphi, \tag{1}$$

which is represented in the given geometry only by the azimuth component A_φ ($\mathbf{e}_\varphi$ is the azimuthal basis vector) and depends on the generalized variable $\eta = \tau - \beta r$, in which the dimensional coordinates of space ρ and time t are replaced by the dimensionless $r = \rho/R_b$ and $\tau = t/T_b$. Here $R_b = (3\mathcal{E}_0/B_0^2)^{1/3}$ and $T_b = R_b/V_0$ are the characteristic scale and time of deceleration of the plasma sphere with the initial kinetic energy of dispersion $\mathcal{E}_0$ in a vacuum magnetic field B_0 [3], the function $f(\eta)$ describes the structure of perturbations, and $f' = df/d\eta$. In accordance with the boundary conditions, on the surface of an ideal moving conductor, we have $A_\varphi = 0$ at $r = a(\tau)$ (a is the radius of the sphere in terms of R_b). Thus, the function $f(\eta)$ satisfies [see (1)] the differential equation

$$f' + \frac{f}{a\beta} - \frac{a^2}{\beta} = 0 \tag{2}$$

under the initial conditions $f(0) = f'(0) = 0$, which follow from the requirement of the continuity of the field $\mathbf{A}$ in the spherical front of perturbations with radius $\rho = V_A t$ ($\eta = 0$), outside of which the field is homogeneous and has the potential $\mathbf{A}(\rho > V_A t) = \mathbf{B}_0 \times \mathbf{e}_\rho \rho/2$.

For a given dependence of the sphere velocity V on time, in particular, for rectangular ($t < 0$ and $V = 0$; $0 < t < T_b$ and $V = V_0 = \text{const}$; and $t > T_b$ and $V = 0$) and quadratic dependences, the form of the function $f(\eta)$ was found in [4, 7]. In our approach, it is required to solve (2) together with the equation of motion of a sphere.

The perturbed field on the sphere $\mathbf{B}_s = \mathbf{B}|_{r=a} = \text{rot}\mathbf{A}|_{r=a}$ determines the magnetic pressure $\mathbf{B}_s^2/8\pi$, in the overcoming which the plasma performs work. With allowance for the integration over the angular dependence of pressure, the latter can be written in terms of $\mathcal{E}_0$ as

$$W \simeq \int_0^a Q^2 a^2 da,$$

where $Q = |\mathbf{B}_s|/B_0$ is the factor of field "amplification" on the magnetic equator of the sphere ($\theta = \pi/2$). In a vacuum field, as is known [3], we have $Q = 3/2 = \text{const}$. The model used takes into account that Q is a function of η in a magnetized background. With allowance for relation (2), it follows from the θ component of the magnetic field at the boundary that

$$Q = \frac{1}{2}\left(3 + \beta^2 \frac{f''}{a}\right).$$

The equation for the rate of variation of the sphere radius in time can be derived in a form similar to the equation in a vacuum field [3]:

$$\frac{da}{d\tau} = \mathcal{F} = \sqrt{1-W} - \frac{2}{3} Q \sqrt{\frac{a^3}{5}}. \tag{3}$$

The first term on the right-hand side of (3) is determined by energy losses because of plasma deceleration. The second term describes the dynamic balance of the gas and magnetic pressures at the moving boundary. After the deceleration ends ($da/d\tau \to 0$), the square of this term characterizes the amount of energy remaining in the sphere and needed to maintain equilibrium.

Equations (2) and (3) lead to the self-consistent dynamic system

$$\frac{df}{d\eta} = -\frac{f}{\beta a} + \frac{a^2}{\beta}, \qquad \frac{da}{d\eta} = \frac{\mathcal{F}}{1-\beta\mathcal{F}}, \qquad \frac{dW}{d\eta} = Q^2 a^2 \frac{da}{d\eta}, \tag{4}$$

the integration of which in the region of variation of the variable $\eta = \tau - \beta a > 0$ allows one to determine the perturbation structure and the law of motion for $\beta < 1$. For $\eta = \eta_0 \to 0$, the system has a singularity, and, therefore, the relations

$$a(\eta_0 \to 0) = \frac{\eta_0}{1-\beta}, \quad f(\eta_0 \to 0) = \frac{\eta_0^3}{(1-\beta)^2(1+2\beta)}, \quad W(\eta_0 \to 0) = \frac{4}{5}\frac{(1+\beta)^2\eta_0^3}{(1-\beta)^5(1+2\beta)^2}$$

are used as the initial conditions (4). The quantity Q is found at each integration step from the algebraic equation

$$(4/3\sqrt{5})\,\beta a^{9/2} Q^2 + 2a^3(1 - \beta\sqrt{1-W})Q = f + 2a^3,$$

derived on the basis of the determination of Q via f'' (see above) with the use of (2) and (3).

The specifics of our approach consists in the simultaneous account of both the energy necessary for maintaining field and plasma equilibrium at the moving boundary and the additional perturbation of the decelerating field at this boundary because of the currents induced in a background plasma. We note that in the papers [8, 10], in which the formulation of the problem is similar to ours, each of the two important factors is considered separately. Below, we illustrate it by a numerical example.

Results of Numerical Simulation. Figure 1 shows the ultimately attainable radius of the sphere a_m on the parameter β. In a sub-Alfvén regime ($\beta < 1$) and in a linear approximation up to $\beta = 0.9$, the background has an insignificant effect on the deceleration radius because of an abrupt dependence of another important parameter on it, namely, the energy $\mathcal{E}_I$ of the dipole magnetic moment which is induced in the cloud and is determined by the surface density of the current on the sphere $(c/4\pi)QB_0 \sin\theta$ (c is the velocity of light). In terms of $\mathcal{E}_0$, this energy can be written as

$$\mathcal{E}_I \simeq \frac{1}{2} Q a^3 \tag{5}$$

[after completion of the transient processes of sphere–medium interaction, $Q \to 3/2$, $a \to a_m$, and $\mathcal{E}_I \to (4/5)\mathcal{E}_0$, which corresponds to the "vacuum" expressions of these quantities]. Variation of $\mathcal{E}_I$ as a_m decreases

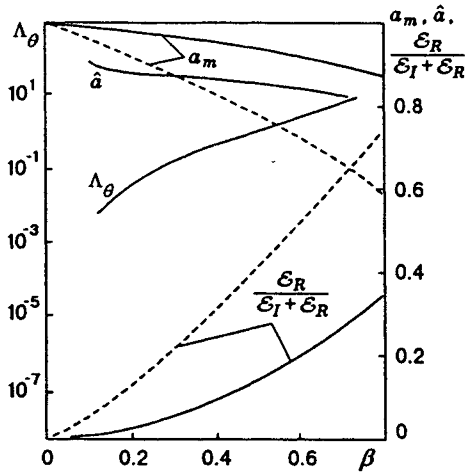

Fig. 1

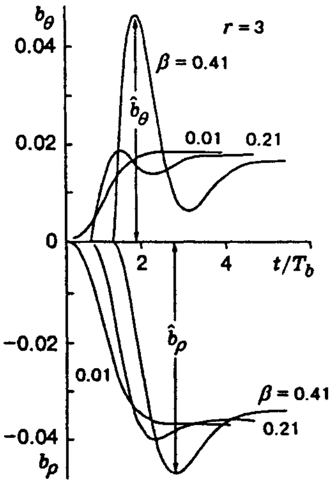

Fig. 2

occurs owing to an increase in energy losses of the cloud for generation of magnetosonic perturbations $\mathcal{E}_R$.

For the radiated MHD energy, the following expression was obtained in terms of $\mathcal{E}_0$:

$$\mathcal{E}_R \simeq \frac{1}{2} \int \frac{\beta^3 (f'')^2}{1 - \beta \mathcal{F}} \, d\eta. \tag{6}$$

Precisely this expression is used in numerical calculations. Integration is performed over the generalized variable $\eta = \tau - \beta a$, together with integration of the above-considered system which describes the field and sphere dynamics. This is taken into account by the kinematic factor $(1 - \beta\mathcal{F})^{-1}$. Figure 1 shows calculated dependences of the relative amount of radiation-induced losses $\mathcal{E}_R/(\mathcal{E}_I + \mathcal{E}_R)$ and a_m on β. For comparison, we give the dashed curves of similar dependences derived in [4] for the models of a sphere which first expands with constant velocity and then instantly stops. It is seen that the method of self-consistent dynamics refines the data on the energetics of the cloud–magnetized medium interaction (by approximately a factor of 2 for $\beta = 0.8$).

Figure 2 shows time dependences of the relative values of the θ and ρ components of magnetic perturbations ($\mathbf{B} = \mathrm{rot}\mathbf{A}$) at the distance $r = 3$ for $\beta = 0.01$, 0.21, and 0.41:

$$b_\theta = \frac{(\mathbf{B} - \mathbf{B}_0) \cdot \mathbf{e}_\theta}{\mathbf{B}_0 \cdot \mathbf{e}_\theta} = \frac{1}{2} \left[\frac{f}{r^3} + \beta \frac{f'}{r^2} + \beta^2 \frac{f''}{r} \right], \quad b_\rho = \frac{(\mathbf{B} - \mathbf{B}_0) \cdot \mathbf{e}_\rho}{\mathbf{B}_0 \cdot \mathbf{e}_\rho} = -\frac{f}{r} - \beta \frac{f'}{r^2}.$$

In the simplified model [4], the amplitudes of the signals are much larger (for example, for $\beta = 0.5$ and $r = 3$, the amplitudes are 6 times larger). With increase in β, the oscillatory character of the signals is manifested increasingly, in contrast to the monotone character in the vacuum limit (see curves for $\beta = 0.01$). This fact is connected with the existence of the quasistatic ($\beta r \ll 1$), transient ($\beta r \sim 1$), and wave ($\beta r \gg 1$) zones of perturbation formation. To check the calculation accuracy, the law of energy conservation was monitored and to do this, we constructed curves of time variation of the work performed by the sphere $W(\tau)$, the energy of the currents induced in the cloud, $\mathcal{E}_I(\tau)$, the radiated MHD energy $\mathcal{E}_R(\tau)$, and the sum $\mathcal{E}_I + \mathcal{E}_R$ for various values of β (for $\beta = 0.8$ in Fig. 3). For convenience, all the curves are reduced to the common origin, namely, the moment of arrival of a signal at the point of observation. The exact equality $W = \mathcal{E}_I + \mathcal{E}_R$ should occur in the limit of completion of the transient process of formation of a radiation field ($\tau - \beta r \gg 1$). It is obvious that in the absence of a medium ($\beta \to 0$), the work of expansion of an ideally conducting body of arbitrary shape with a quiescent center of inertia in a magnetic field $B_0 = \mathrm{const}$ coincides with the energy of the induced currents. The algorithm has an error caused by the error in keeping the indicated balance. This error becomes noticeable with increasing β, but, nevertheless, it does not exceed 5% of the value of W in the range considered, $0 < \beta < 0.9$.

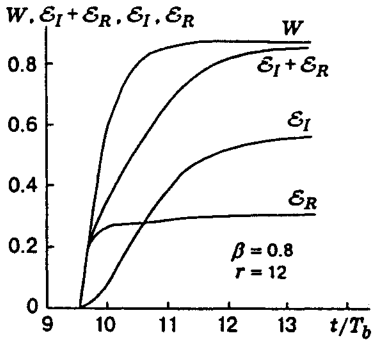

Fig. 3

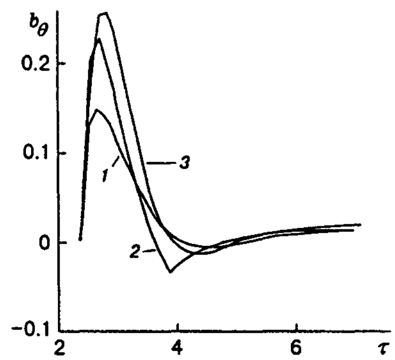

Fig. 4

Figure 4 shows how important is the inclusion of the factors of the pressure and medium-induced field amplification balance on the sphere surface. Curve 1 was calculated by means of the "full" model ($\beta = 0.8$ and $r = 3$), curve 2 was plotted with a decrease in the second term in (3) by a factor of 1000, and curve 3 was constructed for $Q = 3/2 = \text{const}$. Clearly, the simplified models of both types give rise to a pronounced increase in the signal amplitude and, hence, the overestimation of the radiation power.

Recovering the Initial Velocity and Energy of a Plasma Cloud on the Basis of Magnetic Signals. *Quasistatic Method.* If a magnetic probe is positioned in close proximity to the cloud ($\rho \ll V_A T_b$), we may ignore radius-dependent terms, namely, r^{-2} and r^{-1}, in the expressions for perturbations. The thus obtained near-zone approximation holds for $\beta \ll 1/r$, where $r \gtrsim 1$ and $\beta \ll 1$ or for $\beta \ll 1/(r-1)$, where $\beta \lesssim 1$. In this case, to find the relation between the signal characteristics and the dynamic parameters of the cloud, it is possible to use the "vacuum" model into which the magnetosonic model is transformed for $\beta \to 0$. In an ideal MHD approximation, the dynamics of a plasma sphere in a vacuum magnetic field is described by the equation [3]

$$\frac{da}{d\tau} = \sqrt{1 - \frac{4}{5}a^3} - \sqrt{\frac{a^3}{5}}. \tag{7}$$

The signals of the magnetic probe removed from the cloud $\Delta\mathbf{B}(t)$ and $\Delta\dot{\mathbf{B}} = d\mathbf{B}/dt$ are uniquely determined by the sphere dynamics. The derivative with respect to the field

$$\frac{d\mathbf{B}}{dt} = -\frac{1}{2}\frac{da^3}{dt}R_b^3\mathbf{F}, \qquad \mathbf{F} = \frac{1}{\rho^3}[3\mathbf{e}_\rho \cdot (\mathbf{B}_0 \cdot \mathbf{e}_\rho) - \mathbf{B}_0]$$

reaches a maximum at the moment $t = t_*$ when the value of $da^3/d\tau$, which is proportional to the power of energy losses by the cloud, is also maximal. The extremum condition $d^2a^3/d\tau^2 = 0$ gives an equation for the relative radius of the sphere a_* at the moment $\tau_* = t_*/T_b$. We can derive this equation by differentiating Eq. (7) and equating it to zero. As a result, we have $a_* = 0.75$; after substituting this value into (7), we find the instantaneous velocity $da_*/d\tau = \dot{a}_* = 0.53$ in terms of V_0. The characteristic time necessary to reach the maximum loss power is

$$\tau_* = \int_0^{a_*} \frac{da}{\sqrt{1-(4/5)a^3} - \sqrt{a^3/5}} = 0.92.$$

The coefficients a_*, $\dot{a}_*$, and τ_* are constants of the model, which are necessary to define the plasma motion parameters on the basis of magnetic signals.

The limiting radius of deceleration ($R_m = a_m R_b$) is determined by the relation

$$R_m = [-2(\Delta\mathbf{B}_m)^2/(\Delta\mathbf{B}_m \cdot \mathbf{F})]^{1/3},$$

where $\Delta\mathbf{B}_m$ the maximum-in-amplitude field of perturbations, which is recorded by a probe (the "amplitude of saturation").

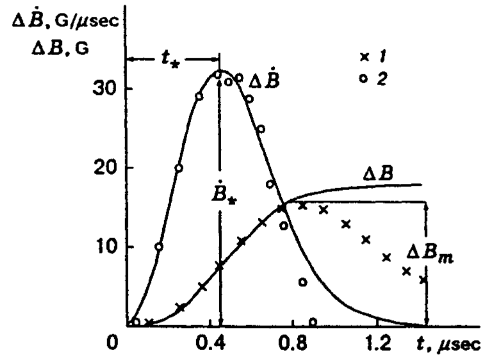

Fig. 5

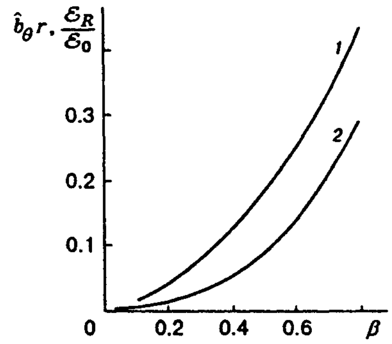

Fig. 6

The thus found effective radius of deceleration R_m can be less than R_b ($a_m < 1$) for two reasons: because of the presence of a background medium as the parameter β increases and a marked penetration of the field into a plasma during deceleration [2]. The latter causes a portion of the converted energy of the directed (radial) motion to convert to the "heat," whose contribution to the full induced magnetic moment can be decreased relative to the contribution of the same amount of energy in the initial form of motion, because this contribution depends on the particular shape of the occurring particle distribution over the densities and velocity components and also on the degree of inheterogeneity of the perturbed field in the cloud [11], i.e., it is determined by the special features of magnetic diffusion. An important example of the nonunique energy redistribution is the phenomenon of polarized jets formed at the end of the deceleration stage in developing the flute instability [2]. As a consequence, the quasistatic energy of the induced current, calculated using the measured "saturation amplitude" $\Delta\mathbf{B}_m$ from the expression

$$\mathcal{E}_I = -\frac{1}{2}\frac{(\Delta\mathbf{B}_m)^2 B_0^2}{\Delta\mathbf{B}_m \cdot \mathbf{F}}, \tag{8}$$

will be smaller [12] that its ideal estimate $(4/5)\mathcal{E}_0$ [3]. The influence of diffusion on the results of analysis can be weakened if one employs an approach effectively applied in processing the data obtained by magnetic probes in experiments [13] on generation of quasispherical clouds of a laser plasma in a vacuum homogeneous field on the KI-1 facility [2]. The approach is based on the fact that at the initial stage from $t = 0$ to $t \sim t_*$, the observed diffusion is small, and the motion occurs in good agreement with the model of a superconducting sphere. From the above relations follow formulas for determination of R_b, the initial energy, and the velocity on the basis of the measured time t_* and the maximum derivative of the signal $\dot{\mathbf{B}}_*$ (for $t = t_*$):

$$R_b = \left(-\frac{2}{9}\frac{t_*}{\tau_*}\frac{B_0^2 \cdot \dot{\mathbf{B}}_*^2}{\dot{\mathbf{B}}_* \cdot \mathbf{F} a_*^2 \dot{a}_*}\right)^{1/3}, \qquad V_0 = -\frac{2}{3}\frac{\dot{\mathbf{B}}_*^2}{\dot{\mathbf{B}}_* \cdot \mathbf{F} R_b a_*^2 \dot{a}_*}, \qquad \mathcal{E}_0 = \frac{1}{3}\frac{V_0 t_*}{\tau_*} B_0^2 R_b^2. \tag{9}$$

Figure 5 shows time dependences of the full amplitude of the field perturbation and its derivative (points 1 and 2, respectively), which were measured by a probe in the plane of the magnetic equator at a distance of 18.6 cm from the laser target located in the field $B_0 = 780$ G ($t_* = 0.45$ μsec and $\dot{B}_* = 33$ G/μsec). Using formulas (9), we find effective values of the parameters $R_b = 6.7$ cm, $\mathcal{E}_0 = 6$ J, and $V_0 = 13.6$ cm/μsec, which were used for calculation of theoretical dependences of the field and its derivative on time by the model of a superconducting sphere in vacuum (curves in Fig. 5). The calculated and measured signals almost coincide up to the moment 0.8 μsec, which is close to $2t_* \sim 2T_b$. Thus, the approximation used holds at the major part of the deceleration stage, which makes it possible to estimate the discrepancy between the energies $\Delta\mathcal{E}_I = \mathcal{E}_I - (4/5)\mathcal{E}_0 = 0.7$ J with the use of the found difference between the real "amplitudes of saturation" and the ideal amplitude equal to 2.5 G (Fig. 5). In the general case, a similar method of determining a portion of energy excluded from the magnetic interaction allows a more precise description of the character of energy transfer between an expanding plasma and an outer magnetized medium.

If the expansion is observed in the background with the known Alfvén velocity V_A, this method yields its effective value $a_m^3\mathcal{E}_0 < \mathcal{E}_0$, which depends on the amount of radiated energy of MHD perturbations: $\mathcal{E}_R = (4/5)(1 - a_m^3)\mathcal{E}_0$, instead of the full magnitude of the initial energy. The quantities $\mathcal{E}_R$ and $\mathcal{E}_0$ are recovered using the found value of $\beta = V_0/V_A$ with the use of the calculated dependences $a_m(\beta)$ (see Fig. 1) and $\mathcal{E}_R/\mathcal{E}_0(\beta)$ (curve 2 in Fig. 6).

Transient-Zone Method. In the transient zone ($\beta r \sim 1$), it is necessary to use the results of the full magnetosonic model for solution of the inverse problem. It is convenient to pay attention to the leading peaks of the θ and ρ signals with amplitudes $\hat{b}_\theta$ and $\hat{b}_\rho$, respectively (see Fig. 2). The peaks correspond to the initial stage of motion at which the effects that are not taken into account in the models and are mainly associated with the development of magnetic diffusion and flute instability are not able yet to be manifested to a considerable extent. This is primarily true for the peak $\hat{b}_\theta$, which is before the peak $\hat{b}_\rho$ on the times (from the moment of arrival of a signal) $t \lesssim T_b$, where, according to the experimental data given above, the shape of the signals is close to ideal. This peak is preferable for recovering the initial velocity of the sphere (the Mach–Alfvén number). The peak of the ρ signal, which decreases as the inverse square of the distance and, therefore, is rather marked only in the transient zone, contains information on the energetics of deceleration. Since it is located on the times $t \lesssim 2T_b$, the degree of its possible distortion in real processes should be clarified by numerical simulation or experimentally. In observations outside the plane of the magnetic equator ($\theta \neq \pi/2$), the following relations for the moment $\hat{t} = \hat{\tau} T_b$ in the peak of the θ signal occur:

$$\Gamma_\theta = 2\hat{b}_\theta + b_\rho = \beta^2 \hat{f}''/r = -U - \beta \hat{f}'/r^2.$$

Here the quantity U is expressed via the measured derivative of the ρ signal with respect to the time dB_ρ/dt. Using the function f, we can write it in the form

$$U = \frac{dB_\rho}{dt}\frac{r}{B_0 V_A \cos\theta} = \frac{db_\rho}{d\eta}\beta r = -\frac{\beta}{r}\left(\frac{\hat{f}'}{r} + \beta \hat{f}''\right),$$

the derivatives of the function f with respect to η being calculated for $\eta = \hat{\tau} - \beta r$; b_ρ is the value of the ρ signal for $\tau = \hat{\tau}$. Combining these relations, it is easy to show that there is a universal characteristic Λ_θ that depends only on β:

$$\Lambda_\theta = \beta^3 |\hat{f}''|^2/|\hat{f}'| = \Gamma_\theta^2/(\Gamma_\theta + U).$$

This characteristic is also determined via the measured amplitude parameters $\hat{b}_\theta$, b_ρ, and U of the magnetic signal. This dependence was calculated (Fig. 1), and this offers the possibility of finding the desired parameter β in experiments with the use of a magnetic probe. The relation $\Lambda_\theta \simeq (2\hat{b}_\theta)^2/|b_\rho|$ holds for large values of the radius ρ, and, hence, signal processing becomes much simpler. This technique is poorly sensitive to measurement errors in $\hat{b}_\theta$ and b_ρ because of the fairly strong manifestation of the dependence of Λ_θ on β.

The characteristic curve of the dependence of the instantaneous relative radius of the sphere $\hat{a}$ on β in the peak of the ρ signal was constructed similarly (see Fig. 1). It is possible to use directly the data of magnetic measurements on the basis of the dimensional equation for the radius ($\hat{A} = \hat{a}R_b$)

$$\hat{A}^3 + \rho^2 \Gamma_\rho \hat{A} - 2b_\theta \rho^3 = 0,$$

where $\Gamma_\rho = \hat{b}_\rho + 2b_\theta$ is an experimentally found characteristic.

In the ρ peak, we have $\Gamma_\rho < 0$ and, as an analysis shows, the equation for $\hat{A}$ has a unique root. If the parameter β is found using the described technique of signal processing in the θ peak, one can determine the dimensionless radius $\hat{a}$ with the use of the curve in Fig. 1. Substituting the values of Γ_ρ and b_θ measured in the ρ peak into a cubic dimensional equation and solving it, we find $\hat{A}$. Finally, based on the relations $R_b = \hat{A}/\hat{a}$ and $\mathcal{E}_0 = B_0^2 R_b^3/3$, one can recover the values of the limiting radius of deceleration and the initial energy of the cloud.

Based on model calculations, we considered examples of such recovery of the initial data. In particular, for the parameters of the direct problem $\beta = 0.8$, $B_0 = 200$ G, $V_0 = 57$ km/sec, and $\mathcal{E}_0 = 18.4$ J ($R_b = 24$ cm), owing to the finite accuracy of its solution (see above) the error of determination of the unknown quantities

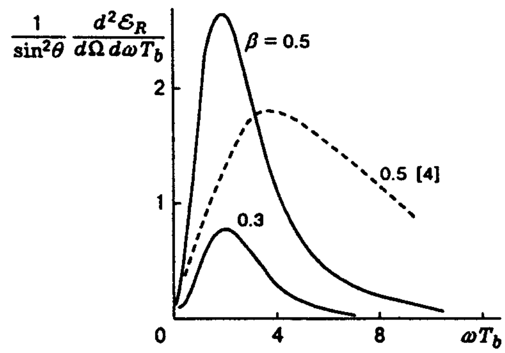

Fig. 7

on the basis of the calculated signals at the point of observation $\theta = 45°$ and $\rho = 72$ cm was approximately 5% for the velocity and 20% for the energy.

Wave-Zone Method. If the probe is located in the wave zone ($\beta r \gg 1$ or $\rho \gg V_A T_b$), based on the recorded time variations in the signal amplitude ΔB_θ decreasing in inverse proportion to the first degree of distance, it is possible also to find the motion parameters β and $\mathcal{E}_0$. In this case, the electrical field of a magnetosonic wave is

$$E_\varphi = -\frac{1}{c}\frac{\partial A_\varphi}{\partial t} \simeq -\frac{V_A}{c}\Delta B_\theta,$$

which corresponds to the power of radiation incident on a unit site

$$\frac{dP}{dS} = \frac{c}{4\pi}\mathbf{E}\times\Delta\mathbf{B} = \frac{V_A}{4\pi}(\Delta B_\theta)^2.$$

The power falling on the entire surface of a sphere of radius ρ has the form

$$P = (2/3)\,(V_A/\sin^2\theta)\,\rho^2(\Delta B_\theta)^2,$$

where θ is the azimuth of probe location. Integration of the squared amplitude of a magnetic signal over the time of observation gives the radiated MHD energy $\mathcal{E}_R = \int P\,dt$.

The ratio $\alpha = \mathcal{E}_R/\mathcal{E}_0$ depends only on β [see (6)]. In the wave zone, the product of the normalized amplitude of the signal in the θ peak by the relative radius of probe location r (see the calculated curve 1 in Fig. 6) is also a function of β: $\hat{b}_\theta r = \xi(\beta)$. By measuring $\hat{b}_\theta = \Delta\hat{B}_\theta/(B_0\sin\theta)$ and $\mathcal{E}_R$, one can find R_b and β with the use of the dependences $\alpha(\beta)$ and $\xi(\beta)$ as the system of two equations, namely, $\hat{b}_\theta\rho/R_b = \xi(\beta)$ and $3\mathcal{E}_R/(B_0^2R_b^3) = \alpha(\beta)$.

It is obvious that this method is most convenient under the full-scale conditions of a homogeneous medium, because it requires measurement of only one component and is less sensitive to the signal shape in comparison with the transient-zone method.

Discussion of Some Restrictions. The limits of applicability of the techniques considered are limited to conditions under which factors that are not taken into account in the magnetosonic model become important. This primarily refers to the degree of magnetization of a medium with respect to the parameter ω/Ω_*, where $\Omega_* = Z_*eB_0/m_*c$ is the gyrofrequency of the background ions and ω is the frequency in the spectrum of cloud-generated perturbations. According to [4], the MHD-radiation spectrum has a maximum for $\omega T_b \sim 4$, and, hence, the quantity $\omega_c = 4/T_b$ acts as a characteristic frequency. We calculated the refined shape of the spectral dependence of the energy emitted to an element of the solid angle $d\Omega$ (solid curves in Fig. 7) in the integration of system (4) for various values of the parameter β from the expression

$$\frac{1}{\sin^2\theta}\frac{d^2\mathcal{E}_R}{d\Omega d\omega T_b} = \frac{\beta^3}{5\pi^2}\left|\int_0^\infty \frac{f''\exp(i\omega\eta)}{\sqrt{1-\beta\mathcal{F}}}\,d\eta\right|^2.$$

The maximum obtained is shifted to $\omega T_b \sim 2$, because the deceleration occurs for the time $\sim 2T_b$, instead of $\sim T_b$, as in the model of [4]. Therefore, the refined value is $\omega_c \sim 2/T_b$, and, as calculations have shown, in practice it does not depend on β. The frequency $4/T_b$ characterizes the upper spectral boundary at its half-height and does not depend on β as well. This spectral characteristic in [4] grows with increasing β (the dashed curve from [4] in Fig. 7 for $\beta = 0.5$) and becomes approximately $10/T_b$ for $\beta = 0.9$.

The approach used corresponds to the zero approximation $\omega/\Omega_* \to 0$ (the frequencies of plasma oscillations of the ions and electrons are assumed to be much higher than the gyrofrequency Ω_*), in which the tensor of dielectric permeability is reduced to a diagonal shape, and the expansion does not occur. Written via the cloud ($\mathcal{E}_0$ and V_0) and medium (B_0, m_*, and Z_*) parameters, the magnetization condition $\omega_c/\Omega_* \ll 1$ is of the form $5 \cdot 10^3 (m_{\rm H} Z_*/m_*)(3\mathcal{E}_0 B_0)^{1/3}/V_0 \gg 1$, where $m_{\rm H}$ is the mass of the hydrogen ion, in the Gaussian system of units. With certainty, this condition is satisfied in the hydrogen background of the near cosmos ($B_0 \sim 0.01$–0.001 G) for typical parameters of, for example, an antiasteroid explosion [6] ($\mathcal{E}_0 \sim 10$–100 megatons and $V_0 \sim 10$–100 km/sec). At the same time, in practice it is difficult to ensure a similar separation of the gyrofrequency and the perturbation spectrum in a laboratory experiment. This indicates the necessity to check the influence of the magnetization parameter $\omega_c/\Omega_* \lesssim 1$ (for example, by means of the hybrid model of [14]) for $\beta < 1$ from the viewpoint of the possible distortion of the shape of magnetic signals and variation of the energy characteristics of the cloud–medium interaction.

Another important restriction is connected also with the parameter ω/Ω_*, and, in addition, it depends on the choice of the point of observation. In the first nonzero approximation relative to the small parameter ω/Ω_*, cloud-generated magnetosonic waves excite Alfvén perturbations in a medium. These waves propagate along the field with the same amplitude [4]. For this reason, the Alfvén component can distort the shape of signals on a fairly large radius ($\rho \gg R_b/\beta$) at the angle of observation $R_b/\rho < \theta \ll 1$ [8]. To decrease the sensitivity to such noise, it is preferable to apply the wave-zone method described above in the equatorial plane. This is more effective, because the perturbation amplitude is maximum for $\theta = \pi/2$ ($\rho =$ const). The lower sensitivity of the transient- and near-zone techniques is connected with the fact that they are used at close distances, where the relative contribution of the magnetosonic mode remains determining.

In concluding, it is noteworthy that the new approaches to recovering the dynamic parameters of an explosive plasma have been developed in the present paper on the basis of the signals of remote magnetic probes under the conditions of magnetized background. The possibility of their effective application in laser-plasma cloud generation experiments in a vacuum homogeneous magnetic field has been shown. In our opinion, the dependence of the relative amount of radiant energy on the Mach–Alfvén number that we have found is an important result, because it refines the estimates of [4]. The magnetosonic model of expansion of a "superconducting" spherical cloud has been calculated with allowance for all the basic factors and with a check of the energy balance for the first time. The position of the maximum and spectral width of the generated perturbations in relative units has been found to be almost independent of the Mach–Alfvén number. One can conclude that these data are not sufficient to find, in particular, the initial velocity of the cloud, and, consequently, an amplitude analysis of magnetic signals is of fundamental importance.

The authors express their gratitude to Yu. P. Zakharov and A. V. Melekhov (the participants of the experiment on the KI-1 facility) for fruitful discussions of the results of magnetoprobing measurements.

This work was performed within the framework of the "Space Experiment in Laboratory" project supported by the Ministry of Science of the Russian Federation within the framework of the state scientific and technical program "Astronomy. Fundamental Space Research."

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